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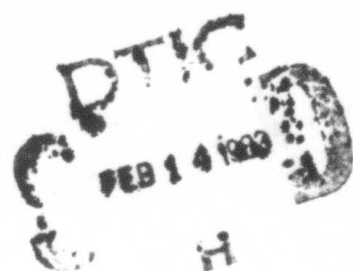
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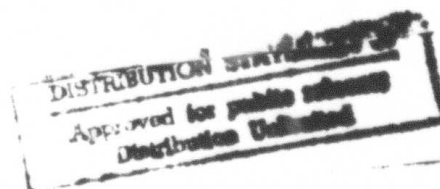
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COMPUTING MOTIONS OF (LOCALLY) PLANAR SURFACES FROM SPATIO-TEMPORAL CHANGES IN IMAGE BRIGHTNESS: A NOTE

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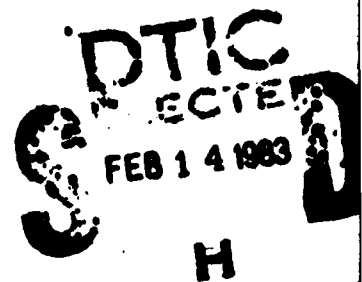University of Maryland
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ABSTRACT

The motion at a locus on the projection surface cannot be obtained locally from spatio-temporal changes in image intensity at that locus. If the imaged surfaces are (locally) planar, however, and generate a set of non-parallel straight line edges in the image, it is possible to obtain estimates of image velocities of a set of image points corresponding to the intersections of the lines. The availability of such a (small) set is sufficient to resolve the underlying optical flow into its translational and rotational components.

1. Introduction

If image motion is to be detected by an (oriented) unit which is small compared to a small (straight line) contour segment (along which the image brightness is approximately constant), the motion of this "oriented point" cannot be determined by a purely local computation. The velocity is constrained by a linear relation but it cannot be completely specified from the information available at the point (Horn and Schunck, 1980; Limb and Murphy, 1975; Marr and Ullman, 1979). The linear constraint on the image motion at a locus (derivable from the relations between the spatial and temporal derivatives of the image brightness function at a point) allows one to compute the direction of motion if its magnitude is known, or the magnitude of motion if its direction is known, but both cannot be obtained from the information available at the given locus (or its immediate neighborhood) at an instant.

Marr and Ullman (1979) convolved the image with a "Mexican hat" operator (Marr and Hildreth, 1979) and computed the local direction of image motion by combining the constraints imposed by a set of zero crossing segments (aggregation of points with zero convolution value). The individual constraints specified the sign of the motion of the segment, i.e., the half plane into which the segment could possibly move. Their method seems to be robust but the degree with which the direction of motion (at an image point) can be constrained by the method depends directly on the orientation distribution of the zero-crossing segments in the vicinity of the point in question.

Horn and Schunck (1980) combined the (linear) constraint
due to motion with a (non-linear) constraint imposed by the
smoothness of the viewed surface, and iteratively minimized
an error function expressing both constraints to obtain a
globally consistent field of image velocities. The scheme re-
quires that the visual field consist of a dense field of image
elements in order to apply the constraints derivable from the
assumption of a smooth (3D) surface. In fact, both the schemes
mentioned are closely related, as they are based on the same
model of relationships between the spatial and temporal deri-
vatives of the image brightness function <NOTE 1>. Both schemes
are local in the sense that only information in the immediate
neighborhood of an image location enters into computations at
that locus (the method proposed by Horn and Schunck (1980) de-
pends on global convergence properties defined over the whole
[enclosed] region) <NOTE 2>. Their major disadvantage is the
requirement that the information be spatially dense in order
to be able to constrain the direction of motion (Batalli and
Ullman, 1980) or to converge to a meaningful solution (Horn
and Schunck, 1980). An alternative to the approach based on
relations between the spatial and temporal derivatives of the
image intensity is to solve the correspondence problem expli-
citly using matching (e.g., Barnard and Thompson, 1980).

2. The constraint of (local) planarity

If the viewed surfaces can be assumed to be (locally at least) planar, and it is possible to extract a set of straight line edge segments in at least one image <NOTE 3>, it is easy to compute an estimate of the image velocities of a number of points without resorting to an iterative, relaxation-like process requiring global consistency. The method is based on the following two observations.

  (a) Under (polar) projection, a straight line remains straight under the motion transformation on the image plane.

  (b) The image velocity component in the direction perpendicular to the orientation of the edge at a point on the edge can be computed directly locally.

We will show next that (b) is sufficient to determine uniquely the image motion of the intersection of two (non-parallel) straight lines. Moreover, because of (a), the motion (in the direction perpendicular to the edge orientation) of an edge as a whole can be neatly estimated by combining the motion of the individual points along the edge (e.g., by linear least squares fitting). The whole scheme can be regarded as an example of local/global interaction where a global operation (edge finding) is used to focus attention on places where further information gathering (motion detection) can be performed with reasonable accuracy.

2.1 Computing the component of edge motion perpendicular to the edge

As mentioned above, the image displacement at a point cannot be computed locally without introducing additional constraints. Such local computation is possible, however, along a given image direction. The problem is reduced then to a one-dimensional problem of estimating the displacement of a known one-dimensional image intensity distribution. (It is assumed here that the distribution does not change significantly from frame to frame, i.e., that the change in surface orientation due to the surface motion has only a negligible effect on the image brightness due to the small inter-frame distance). The displacement (d) of an edge at an image point can easily be found using, e.g., the Taylor expansion about the given point

(1) $$d = \{ I_\ell + \sqrt{(I_\ell^2 + 2I_{\ell\ell}\Delta I)} \} / I_{\ell\ell} + \ldots$$

where I_ℓ is the first spatial derivative of the image intensity along the direction ℓ (perpendicular to the edge orientation), $I_{\ell\ell}$ is the second spatial derivative along the same direction, and ΔI is the first time derivative of the image intensity at the given point. (Under certain circumstances one can discard all higher order terms in (1), as was done, e.g., in Fennema and Thompson (1978) or Horn and Schunck (1980).) Because a straight line remains straight through its displacement, i.e., all points have to lie on a straight line after the displacement, (it is assumed that the image intensity along the edge does not

vary significantly (at least within the extents of measured
displacements)), the (perpendicular) displacements at all
points along the edge are related by a linear relation. We
can thus use, e.g., linear least squares fitting or its
modification, the RANSAC method (Fischler and Bolles, 1980)
to obtain an estimate of edge motion from a set of local
measurements. The planar motion of a (straight) line can
be conceptualized as a translation (its points all move with the
same velocity) and a rotation (its points all move with the
same angular velocity). This last condition constitutes the
linear relation mentioned above; the displacement at a point
on the edge is a function of the distance from some fixed, but
arbitrary point on the edge (see Figure 1).

2.2 Determining the motion of an intersection of two straight lines

When two straight lines intersect and their motion in the
direction perpendicular to their orientation is known (the mo-
tion of the line as a whole can be summarized by two parameters:
the translation of an arbitrary point on it, the "origin", and
its angular speed) the image motion of their intersection can
be determined easily. This is illustrated in Figure 2. The
displacements of the intersection X in the directions perpendi-
cular to the orientations of the two edges are known. It follows
that the intersection has to move in the two directions simul-
taneously. Algebraically, the velocity of X can be obtained

by solving two linear equations in two unknowns:

(2) $\underline{dx}.\overline{d1}=d1$ and $\underline{dx}.\overline{d2}=d2$

Here d1 and d2 (the velocity components of $\underline{dx}$ in the directions of $\overline{d1}$ and $\overline{d2}$) are known as well as the directions $\overline{d1}$ and $\overline{d2}$ (perpendiculars to the edges). $\underline{dx}$ is the (2D) velocity vector specifying the motion of the intersection X.

3. Discussion and conclusion

Given a set of straight lines, a set of estimates of image velocities of their intersections can be obtained using the method described above. The accuracy of the velocity estimates depends not only on the quantization errors but also on the lengths of the lines; the longer the lines the more accurate the estimates will be. The intersections do not have to lie in the "visible" part of the image plane to be useful.

The analysis presented here offers a simple but powerful method to obtain estimates of image velocities in the case when the (3D) surfaces are planar and contain a set of straight line edges (as is usual in man-made environments). A has been shown (Prazdny, 1981), a small set of image velocities is sufficient to obtain the parameters of (relative) motion <NOTE 4> of the surface causing the image velocities. (The method employed is a minimalization computation relatively insensitive to noise).

NOTES

<NOTE 1>.

This can be seen by observing that the relation between the image motion and the spatial and temporal derivatives of the image brightness function can be expressed (under certain conditions; see the text) as

$$\partial I/\partial t = \underline{v}.\nabla I$$

where $\underline{v}$ is the image velocity vector, and $I(x,y,t)$ is the image brightness function. Using the convolution values $S(x,y,t)= \nabla^2 G * I(x,y,t)$ ("$\nabla^2 G *$" denotes the convolution operation with the "Mexican hat" operator), the same relationship is expressable as

$$\partial S(x,y,t)/\partial t = \underline{v}.\nabla S$$

The advantage of this second approach is that it guarantees smoothness (and thus differentiability) even in regions of sharp image brightness changes.

<NOTE 2>.

Fennema and Thompson (1978) report a scheme based on the equations used also by Horn and Schunck (1980). Instead of a relaxation-like scheme, however, they used a Hough space approach to minimize the effects of noise. Their scheme is restricted to object motions not containing rotations or looming.

<NOTE 3>.

The analysis here is restricted to two temporally proximal images. One of the images is arbitrarily chosen to be the "first" image on which the line finding process operates.

<NOTE 4>.

Once the focus of expansion specifying the translational component of the relative motion is computed, the directions of the rotational and translational image velocity components at each point in the image are known. All information about the relative

depth is contained in the magnitude of the translational component of the image velocity. If we know the image velocity component perpendicular to the orientation of the "oriented point" at an image locus we can project it onto the direction of the translational component (along the direction specified by the rotational component) to obtain the magnitude of the translational component (and thus information about relative depth).

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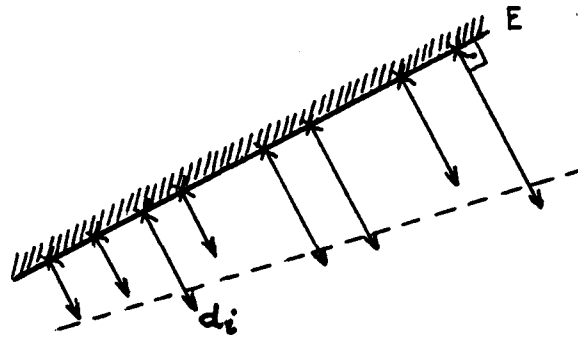flow generated by a curvilinearly moving observer
CGIP (in press)

Figure 1

Using equation (1), the displacement (di) in the direction
perpendicular to the edge E at a number of locations
along the edge (x) can be computed locally. These local
measurements are then combined to obtain a global charac-
terization of the edge motion by fitting a straight line
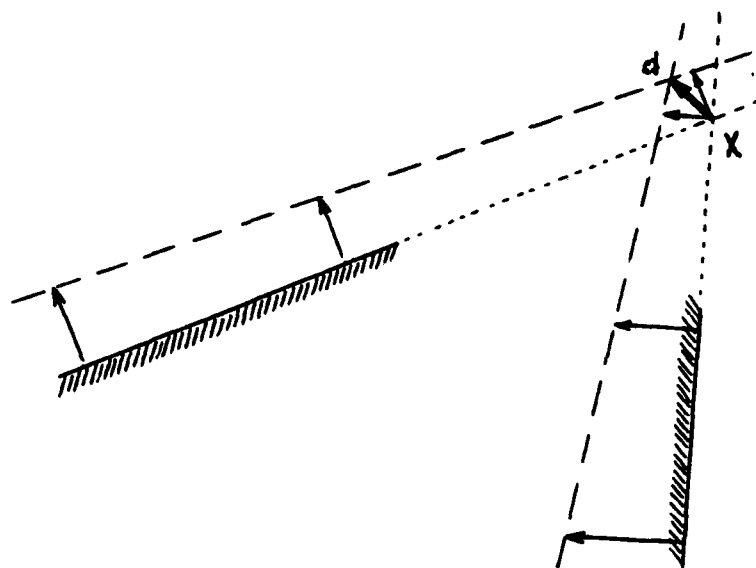(dashed line) to these displacements.

Figure 2

The displacement (d) of the intersection of two lines is deter-
mined completely if the motions of the two lines in the direc-
tions perpendicular to their orientations are known.

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